NONLINEAR WAVE MODULATION IN AN INVISCID AND VISCOUS FLUIDS-FILLED THIN VISCOELASTIC TUBE

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DEDICATION

This work is humbly dedicated to all my valuable treasures in life:

To My Supervisor,
Dr. Choy Yaan Yee
Thank you so much for always be patience and guided me well until this thesis was complete.
To My Beloved Parent,
Mak and Ayah
Who always picked me up on time, making me be who I am and encouraged me to go on every adventures.
To My siblings,
Kak Yah, Kak Ti, Be Ing, Be Din, Cik Ann, Be We, Pikah and Pell
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To My Friend,
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ABSTRACT

This research is aimed to study the modulation of nonlinear wave in an artery filled with blood. In recent years, many researchers have carried out studies on arterial wave modulation with different perspectives related to blood flow. Most of the studies focused on wave modulation in the artery by considering the artery as thin elastic tube. The studies of wave modulation in the artery by treating the artery as thin viscoelastic tube are rather limited in the literature. Therefore, in this research, the artery is considered as an incompressible, prestressed, thin walled viscoelastic tube. By considering the blood as an incompressible inviscid fluid and viscous fluid, two mathematical models of nonlinear wave modulation in thin viscoelastic tube are formed through the use of the reductive perturbation method. The equation of fluids used in this research are exact equations where boundary condition of fluid under consideration. It is shown that the governing equation for the model of inviscid fluid flow in viscoelastic tube is nonlinear Schrödinger equation (NLSE). The governing equation for the model of viscous fluid flow in viscoelastic tube is dissipative nonlinear Schrödinger equation (DNLSE). By seeking the progressive wave solutions to the NLSE and DNLSE, it is observed through the graphical outputs that the solutions of NLSE and DNLSE admit the downward bell-shaped wave with various amplitude. Besides that, based on the solution of NLSE, as the value of space increases, a constant width of wave is obtained with different depths and the resistance pushing through the blood flows in artery is existed. Whereas the solution of DNLSE showed that the solitary wave formed a steep wave profile as it propagates. In addition, the effects of radial velocity, axial velocity, pressure of tube and hydrostatic pressure on the blood flow characteristics are shown graphically.

ABSTRAK

Kajian ini bertujuan untuk mengkaji modulasi gelombang tak linear dalam arteri yang dipenuhi dengan darah. Dalam beberapa tahun kebelakangan ini, ramai penyelidik telah menjalankan kajian mengenai modulasi gelombang arteri dengan perspektif yang berbeza yang berkaitan dengan aliran darah. Kebanyakan kajian tertumpu pada modulasi gelombang dalam arteri dengan mempertimbangkan arteri sebagai tiub elastik nipis. Kajian modulasi gelombang dalam arteri dengan menjadikan arteri sebagai tiub viskoelastik nipis agak terhad dalam kajian literatur. Oleh itu, dalam kajian ini, arteri dianggap sebagai tiub viskoleastik yang tidak dapat dimampatkan, prategasan, tipis. Dengan mempertimbangkan darah sebagai bendalir tidak likat dan bendalir likat yang tidak dapat dimampatkan, dua model matematik untuk modulasi gelombang tak linear dalam tiub viskoelastik nipis dibentuk melalui penggunaan kaedah perturbasi reduktif. Persamaan bendalir yang digunakan dalam penyelidikan ini adalah persamaan tepat dimana keadaan sempadan bendalir dipertimbangkan. Telah ditunjukkan bahawa persamaan yang diperolehi oleh model aliran bendalir tidak likat dalam tiub viskoelastik adalah persamaan Schrödinger tak linear (NLSE). Persamaan yang diperolehi untuk model aliran bendalir likat dalam tiub viskoelastik adalah persamaan Schrödinger tak linear yang disipatif (DNLSE). Dengan mencari penyelesaian gelombang progresif kepada NLSE dan DNLSE, ia dapat diperhatikan melalui output grafik bahawa penyelesaian NLSE dan DNLSE telah membentuk gelombang berbentuk loceng arah ke bawah dengan pelbagai amplitud. Di samping itu, berdasarkan penyelesaian NLSE, oleh kerana nilai ruang bertambah, lebar gelombang tetap dapat diperolehi dengan kedalaman yang berbeza dan terdapat rintangan yang mendorong melalui aliran darah dalam arteri. Sedangkan penyelesaian DNLSE menunjukkan bahawa gelombang solitori membentuk profil gelombang curam ketika ia tersebar. Tambahan pula, kesan halaju radial, halaju paksi, tekanan tiub dan tekanan hidrostatik pada ciri aliran darah telah ditunjukkan secara.

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LIST OF SYMBOLS AND ABBREVIATIONS

a	-	Amplitude of solitary wave
c.c.	-	Complex conjugate of the relating quantities
c_0	-	Wave speed of Moens-Korteweg
E	-	Energy of the particle
e_r	-	Unit base vector in radial component
e_θ	-	Unit base vector component in circumferential
e_z	-	Unit base vector in axial component
g	-	Scale parameter
H	-	Initial thickness of membrane
h	-	Thickness after static deformation
h'	-	Final thickness of membrane
I_1	-	First invariant of Finger deformation tensor
i	-	Imaginary number
K	-	Coefficient of elastic
$[K^{(q)}]$	-	Normalized load stiffness matrix
k	-	Wave number
L	-	Length of blood vessel
m	-	Mass of tube
n	-	Exterior unit normal vector
P_0	-	Uniform inner pressure
P^*	-	Dimensional fluid pressure in the tube surface
P_r^*	-	Dimensional fluid reaction force
$\overline{P}$	-	Dimensional pressure function of fluid
p	-	Non-dimensional fluid pressure in the tube surface
$\overline{p}$	-	Non-dimensional fluid pressure function

p_r	-	Non-dimensional fluid reaction force
p_w	-	Mass of the tube density
$\{Q\}$	-	Normalized load vector
q	-	Non-dimensional axial fluid velocity
R	-	Radius of blood vessel
R_0	-	Radius at the origin of the coordinate system
r	-	Position vector after final deformation
r_f	-	Radius of final mean
r_0	-	Radius of the origin after static deformation
T	-	Vector tangent
$\lvert T \rvert$	-	Length of vector tangent
T_1	-	Membrane force along meridional curve
T_2	-	Membrane force along circumferential curve
$\mathbf{t}$	-	Unit tangent vector along the deformed meridional curve
t	-	Non-dimensional time parameter
t^*	-	Dimensional time parameter
t_{kl}^*	-	Cauchy stress tensor
t_{11} and t_{22}	-	Stress components
U	-	Nonlinear evolution equation
U^*	-	Complex conjugate of U
u	-	Non-dimensional radial displacement
u^*	-	Dimensional radial displacement
$u^*\left(z^*,t^*\right)$	-	Finite time-dependent displacement component in radial direction
V	-	Flow velocity of fluid
V^*	-	Axial velocity component of the fluid
V_r^*	-	Dimensional radial direction of fluid velocity
V_z^*	-	Dimensional axial direction of fluid velocity
v	-	Non-dimensional radial fluid velocity
$W\left(z^*,t^*\right)$	-	One-dimensional longitudinal coordinates field
x	-	Axial coordinate

Z^*	-	Dimensional axial coordinates of a material point in the nature state
z	-	Non-dimensional axial coordinates after static deformation
z^*	-	Dimensional axial coordinates after static deformation
α	-	Material constant
β_i	-	Functions of the starting alter form
δA	-	Deformation of vessel
ε	-	Small parameter
η	-	Coefficient of viscous
Λ_z	-	Arclength of the corresponding curve
Λ_θ	-	Arclength along the circumferential curve
λ	-	Group velocity
λ_θ	-	Initial deformation in the circumferential direction
λ_z	-	Initial stretch in axial direction
λ_1	-	Stretch ratio along the meridional curve after final deformation
λ_2	-	Stretch ratio along the circumferential curve after final deformation
μ	-	Kinematic viscosity of fluid
μ_1	-	Coefficient of dispersive term
μ_2	-	Coefficient of nonlinear term
μ_3	-	Coefficient of dissipative term
μ_v	-	Dynamic viscosity of the fluid
$\bar{v}$	-	Non-dimensional fluid kinematic viscosity
ν	-	Non-dimensional kinematical viscosity
ω	-	Angular frequency
Φ	-	Tapering angle
φ	-	Coefficient of viscoelasticity
ψ	-	Wave function of the particle
ρ	-	Density of blood
ρ_0	-	Mass density of the tube after finite static deformation
ρ_f	-	Mass density of fluid
Σ	-	Strain energy density function of the membrane

σ_{11} and σ_{22} - Nondimensionalized principal stress

σ_t - Approximated relation function

τ - Temporal variable

θ - Circumferential direction in spatial configuration

ξ - Spatial variable

LIST OF APPENDICES

LIST OF PUBLICATION

1 Normaisarah C. Ahmed, Y. Y. Choy and T. K. Gaik (2017). Nonlinear wave modulation in thin viscoelastic tube filled with inviscid fluid, *AIP Conference Proceeding*, 1974 (020088), American Institute of Physics, pp. 1-8. (Published indexed in SCOPUS)

CHAPTER 1

INTRODUCTION

1.1 General introduction

The blood flow in the human body has become a topic of interest in anatomy among researchers. Even if its significance is not well spoken of, there was an important question in the history of anatomy and physiology. Around 1578 to 1657, William Harvey became the first researcher to describe the blood's circulation in the body (Ribatti, 2009). Harvey learned about the human body by dissection and anatomical observation through direct experiments on the hearts and circulatory systems of animals. Working from that point made him understand the role of valves in controlling blood flow, the place of pulmonary arteries, and the fact that the heart does not heat the blood but instead pumps it into the arteries. In 1628, Harvey's work 'Exercitatio Anatomica de Motu Cordis et Sanguinis in Animalibus' or also known as 'On the Motion of the Heart and Blood' was published in Frankfurt (Schultz, 2002).

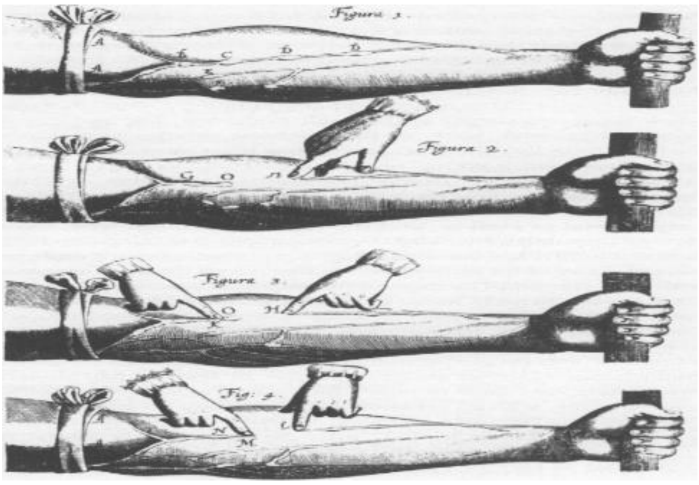

Figure 1.1: An experiment from Harvey's *Exercitatio Anatomica de Motu Cordis et Sanguinis in Animalibus* that illustrated the venous valves (Schultz, 2002).

Based on Figure 1.1, Harvey replicated what Fabricius had done and tied a ligature around the upper part of a man's arm. He observed that when the ligature was tied tightly, the flow of blood from arteries and veins was cut off, but when loosened slightly, the blood from the arteries is allowed to flow into the lower part of the arm but was unable to escape back to the upper part. Harvey recognised this when the veins became more visible which means that the blood is full. As the veins swelled, he also noticed small bumps in the veins which were the valves discovered by Fabricius. If someone took a finger and sought to force the blood away from the shoulder towards the hand, it would quickly become clear that the blood is unable to flow in that direction. The valves in veins prevent the backward flow of blood.

Blood flows in one direction throughout the body and it is in the lungs that the transformation of venous blood to arterial blood takes place. The blood from the heart is carried away by blood vessels named arteries. There are two arteries that do not carry oxygenated blood, which are the pulmonary artery and umbilical arteries. Arteries can be divided into subcategories such as large arteries, medium arteries, arterioles and capillaries. Blood flowing in large arteries is considered following standard behaviour which means that it is a Newtonian fluid, while blood flowing in medium arteries is of non-standard behaviour which means that it is a non-Newtonian fluid (Bessonov *et al*., 2016).

A model of blood flow can be developed by assuming that the cross-section area of a blood vessel remains the same with time. Another assumption for the cross-section area is that it is constant over distance, while the pressure gradient is also assumed to be constant over the distance (Rahman & Haque, 2012):

$$\frac{dQ}{dt}+\frac{4\pi\mu}{S}Q+\frac{S}{2\rho}\frac{dP}{dz}=0, \tag{1.1}$$

where ρ is the density of blood, Q is the initial value, $\frac{dP}{dz}$ stands for the gradient of pressure, and μ is the kinematic viscosity of blood.

Poisuelli's equation is taken into consideration when building the mathematical model for blood pressure. Both blood flow rate and pressure are determined using Poisuelli's equation which is given as:

$$Q=\frac{\pi R^4}{8L\mu}P, \tag{1.2}$$

where L is the length and R is the radius of the blood vessel.

Among the researchers who had extensively studied arterial wave mechanics is Hilmi Demiray. In 1998, Demiray and Dost presented their work about the propagation of solitary waves in a prestressed thick walled elastic tube filled with an incompressible inviscid fluid. Due to the dependence of the coefficients of the Korteweg-de Vries (KdV) equation on the initial deformation, material parameters and thickness ratio, it was observed that the solution profile changed with these parameters. The result indicated that for biological tissues, the wave profile is not so sensitive to changes in thickness, and that material nonlinearity is more important than geometrical nonlinearity.

In 2001, Demiray presented his research on nonlinear waves propagation in viscous fluid contained in a viscoelastic tube. The reductive perturbation method was also involved in the long-wave approximation in order to propagate small-but-finite amplitude waves in the prestressed viscoelastic tube. This method that governed this research obtained the master equation needed to find various evolution equations. Then, some special cases had occurred which led to a reduction of the evolution equations to the Burgers' equation, KdV equation, KdV Burgers' (KdVB) equation, perturbed Burgers' equation, perturbed KdV equation and perturbed KdVB equation. In 2004, Bakirtas and Demiray introduced a paper on the amplitude modulation of nonlinear waves where the artery is treated as a tapered, long and thin-walled elastic tube. The blood was examined as an incompressible inviscid fluid. By using the reductive perturbation method, the nonlinear evolution equation was governed as a nonlinear Schrödinger (NLS) equation with a variable coefficient.

Afterwards, Demiray presented a paper on nonlinear wave propagation in fluid-filled thin viscoelastic tubes as a weakly dispersive case with an incompressible inviscid fluid. He employed the reductive perturbation method to observe the propagation of small-but-finite amplitude waves; evolution equations such as the modified forms of the Burgers', the KdV and the KdVB equations were obtained (Demiray, 2005a). Subsequently, Demiray proposed in his paper on the nonlinear wave modulations in a thin-walled prestressed elastic tube with a variable cross-section. The fluid was assumed to be an incompressible viscous fluid. By employing the reductive perturbation method, the dissipative NLS equation with variable coefficients was obtained. The results obtained showed that for narrowing tubes, the wave speed increases with increases in the distance parameter τ, while for expanding tubes, the wave speed decreases as the distance parameter τ increases (Demiray, 2006).

1.2 Background of the problem

The study of arterial blood flow has great significance in solving complex problems in science and technology. Demiray continued his work by utilising a prestressed thin elastic tube for the modulation of non-linear waves in a viscous fluid contained in an elastic tube. He utilised the reductive perturbation method to obtain the dissipative NLS equation (Demiray, 2001b). Next, Akgun presented the amplitude modulation of waves in an elastic tube filled with a layered fluid that emphasised a prestressed thin elastic tube. The outer layer of fluid was assumed to be inviscid, whereas the cylindrical core was considered to be viscous. The reductive perturbation method was used and the governing evolution equation was obtained in the form of the dissipative NLS equation (Akgun, 2004). In 2009, Demiray proposed the propagation of weak nonlinear waves in a prestressed thin elastic tube. Blood was considered an incompressible heterogeneous fluid with variable viscosity. By employing the reductive perturbation method, an evolution equation was obtained in the form of the perturbed Kortewe-de Vries equation.

In recent years, arterial wave modulation in blood flow has become a problem of interest amongst researchers. Many studies concentrate on the propagation of waves in arterial walls instead of wave modulation. On the other hand, not many researchers have studied on nonlinear wave modulation in a thin-walled viscoelastic tube with two different fluids which are inviscid fluid and viscous fluid.

1.3 Problem statement

In recent years, many researchers have been involved in the study of blood flow in arteries. Blood flow in arteries was dominated by the unsteady flow phenomena. The cardiovascular system is an internal flow loop with multiple branches. Arteries are living organs that can adapt and change according to varying hemodynamic conditions. In general, human blood is known to be an incompressible non-Newtonian fluid with variable viscosity, while arteries are assumed to resemble thick viscoelastic tubes (Holzapfel *et al*., 2002). However, studies on blood flow in a thin viscoelastic tube have yet to be done in detail. Thus, in this research, the artery is considered as an incompressible and prestressed thin walled viscoelastic tube, while the blood that circulates in the artery is assumed to be an incompressible inviscid and viscous fluid.

In a previous study, Yuan and Tao studied the movement of nonlinear waves in a fluid-filled thin viscoelastic tube with an incompressible inviscid fluid; however, the equation of inviscid fluid is an approximate equation. Besides, the type of solution for field equations did not require the use of imaginary numbers to obtain the solution to the KdV equation (Yuan & Tao, 2010). As in this research, the equations of inviscid fluid and viscous fluid were stated as exact equations and an imaginary number will be added in the calculation of field equations. This study adds to the knowledge in this field as no wave modulation using a thin viscoelastic tube filled with viscous fluid has yet been done to date.

Based on this research, some issues had arisen in the process of structuring this study such as the assumptions needed for the modelling of the artery and blood, the method to be used, and the governing equation to be chosen for the corresponding mathematical model. For this research, the progressive wave solution will be calculated using the MATLAB software.

1.4 Research objectives

The objectives of this study are to:

i. derive a nonlinear wave modulation evolution equation to represent a prestressed thin viscoelastic tube filled with an inviscid fluid using the reductive perturbation method.
ii. derive a nonlinear wave modulation evolution equation to represent a prestressed thin viscoelastic tube filled with a viscous fluid using the reductive perturbation method.
iii. obtain the progressive wave solutions for the nonlinear evolution equations in (i) and (ii).

1.5 Scope of study

This study focuses on the wave modulation in an artery. The blood is assumed to be an incompressible inviscid and viscous fluid. The artery is represented in this study by a prestressed thin-walled viscoelastic tube. Then, the dimensional equations of fluids and tube will be changed into non-dimensional equations. By using the reductive perturbation technique, various orders of differential equations will be obtained. Next, the solutions to these various orders of differential equations will be obtained in order to set the nonlinear evolution equation. After that, the nonlinear evolution equation is solved analytically using the progressive wave solution.

1.6 Significance of study

The interface between mathematics and biology has started to cultivate new mathematical areas. Various parts and ideas from mathematics and biology work together to produce enhanced results. This research presents a mathematical model of nonlinear wave modulation in an arterial artery. The finding of this research will benefits researchers interested in uncovering critical areas in the medical field related to wave modulation in an artery. In addition, this study can be used by anyone who is interested to do research in several related areas such as mathematics, sciences, biology, fluid dynamics and others.

Failure of the blood vessels may lead to several diseases such as cardiovascular disease. In this research, two mathematical models for nonlinear wave modulation in blood flow along arterial artery are exhibited. The outcome of this study is to determine whether the blood flow patterns can be utilized to identify the early stage of the arterial disease. Moreover, in future, any researchers can refer the results of this study to validate the calculation processes of inviscid equation and viscous equation.

1.7 Outline of thesis

This thesis intends to study the wave modulation in a prestressed thin viscoelastic tube using two different fluids which are an inviscid fluid and a viscous fluid. The researcher will thus give an overview of the main contents of the thesis to help organise any thoughts or arguments made along the journey of this research. Including this introductory chapter, there are five chapters in this thesis. Each main chapter consist of several sub-chapters to make the thesis more appropriate.

The first chapter is an introductory chapter on the arterial blood flow in the human body. In this chapter, some of Demiray's papers related to the viscoelastic tube were discussed. Besides that, this chapter also consisted of the background of the problem, the problem statement which states the assumptions of this research, objectives of the research, scope of the study, significance of study, methodology that will be used in this research, and an outline of the thesis.

Chapter 2 begins with the histories of solitary waves, the KdV equation and the NLS equation. These sub-topics also involve the general forms of the KdV equation and the NLS equation together with their properties and applications in various fields. Next, the analytical solution for the NLS equation and the reductive perturbation method will be presented. The following sub-topics provide some brief introduction and explanation on the solution for the NLS equation, the reductive technique, the perturbation technique and the reductive perturbation technique. Other than that, wave propagation and wave modulation in elastic and viscoelastic tubes will also be elaborated in this section. This part covers studies conducted from 1983 to 2019. These studies' findings and results will be discussed in this sub-topic. Lastly, the tube equation that will be used in this mathematical modelling will be elaborated.

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